PRECISE TUNE MEASUREMENTS FROM MULTIPLE BEAM POSITION MONITORS*

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Abstract

One of the main limitations for precise tune measurements using kicked turn-by-turn data is the beam decoherence, which can limit the available signal to a reduced number of turns. Applying Laskar's frequency analysis, on measurements from several beam position monitors, a fast and accurate determination of the real tune is possible. The efficiency of the method is demonstrated when applied in turn-by-turn data from the ESRF storage ring and CERN's Super Proton Synchrotron. Estimates from tracking simulations and analytical considerations are further compared with the experimental results.

INTRODUCTION

The frequency map analysis method [1] is a well–established technique of nonlinear dynamics that has already been successfully applied to theoretical [2] and experimental [3, 4] beam dynamics studies. The method permits the fast and accurate determination of the fundamental frequencies (tunes) of single particles (or beam centroids) by the numerical treatment of tracking data (or position measurements) around the ring. In the case of beam position measurements and in the presence of tune dependence with the amplitude or the momentum (chromaticity), the decoherence [5] of the beam diminish drastically the number of turns for the tune determination, with data above the noise level. A way to overcome this problem is to use measurements from multiple beam position monitors (BPMs), which are placed around the ring. In principle, this approach will allow the precise determination of tunes after only a small number of turns, before the beam decoheres. A preliminary study following this approach has already been done for the European Synchrotron Radiation Facility (ESRF) storage ring [4].

In the present paper, we study in detail the problem of tune determination by analyzing measurements from multiple BPMs, both theoretically and experimentally. First, using a theoretical "ideal" model of the ESRF storage ring, we study the convergence to the real tune values of the tunes computed from data acquired from multiple BPMs, in the absence of decoherence. Then, we turn our attention to real experimental data from multiple BPMs and study the effect of decoherence on the tune determination.

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THEORETICAL TREATMENT

As a toy model, we consider an ideal, symmetric lattice for the ESRF storage ring. The ideal ESRF lattice has a 16–fold periodicity and in every super-period 14 BPMs are placed. In the simulations, a symplectic integrator [6] is used for the tracking, while the numerical analysis of the data and the tune determination is performed by the TRIP package [7].

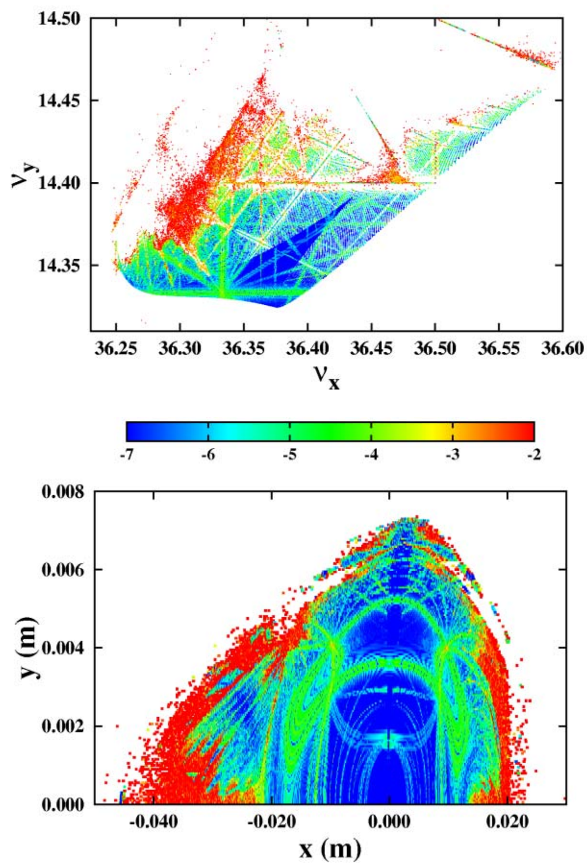

Figure 1: Frequency map for an ideal lattice of the ESRF. The initial conditions are taken over a mesh in the horizontal (x) and the vertical (y) direction (bottom), and the corresponding frequencies are plotted in the frequency space (top). Each point is colored according to the color code associated to the values of the diffusion index D.

The frequency map of the system (Fig. 1) is constructed by the analysis of data acquired from only one BPM which is located in the middle of a straight section of the accelerator. As a stability index we use the tune variation with time.

In particular, we compute the horizontal ν_x and the vertical ν_y tunes over two successive time spans of 600 turns and define the diffusion index D as the logarithm of the root mean square of the tune differences. This diffusion rate is colored by a color scale from blue for regular orbits to red for chaotic ones.

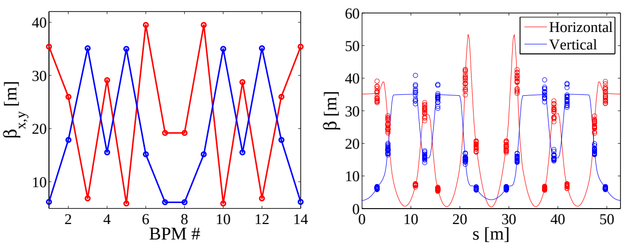

Figure 2: Beta functions from the ideal ESRF model (left) and as measured in the real storage ring (right).

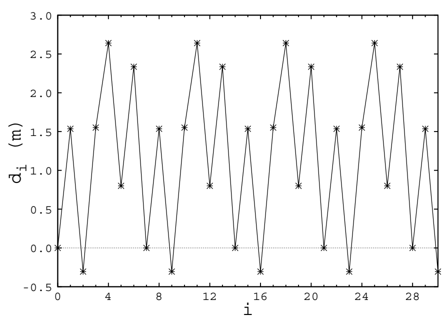

Figure 3: The distance d_i between fictitious (equally spaced) and real BPMs around the ESRF ring, as a function of the BPM index i. The line $d_i = 0$ is also plotted.

Since the ESRF ring has a 16–fold periodicity, there exist 14 families each one containing 16 symmetric BPMs, with respect to the ring optics. The azimuthal distance between successive BPMs of the same family is constant and equal to the length of the superperiod. The 224 BPMs existing around the ring are neither equally spaced and their optics differ, as plotted in Fig. 2a. In Fig. 3 we plot the distance d_i between the positions of 224 fictitious, equally spaced, BPMs around the ring and the real BPMs, with respect to the index i of each BPM. The $i = 14$ periodicity resulting from the 16–fold periodicity of the ring is evident. Obviously, higher order periodicities are also present e.g. $i = 28 = 2 \times 14$. Additionally, the distance between every 7th BPM is constant, due to the fact that each superperiod of the ring has an additional internal mirror symmetry, although these BPMs do not have the same optics (see Fig. 2a).

For demonstrating how data acquired from multiple BPMs are used to accurately determine the real tunes in less turns, we consider an on–momentum beam in the stable region of Fig. 1, near the ideal closed orbit with initial conditions $x = y = 1mm$. The real tunes ν_x, ν_y of the orbit are computed by the frequency analysis of measurements from only one BPM for 10^4 turns. In Fig. 4, we present the convergence rates to the real tunes of the ones

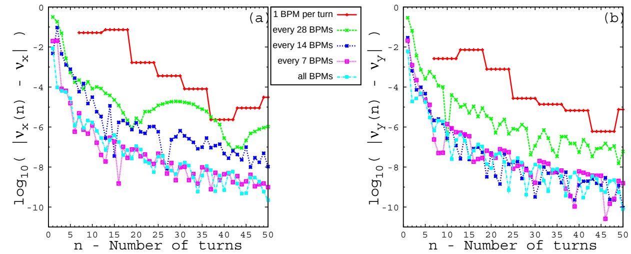

Figure 4: Convergence of the (a) horizontal ν_x and (b) vertical ν_y frequencies to the tune values extracted by a standard turn by turn analysis as a function of the number of turns n. The following schemes were used: data from only one BPM per turn (red curves), averaging of tunes computed from: 1) 28 families of 8 symmetric BPMs (green curves), 2) 14 families of 16 symmetric BPMs (deep blue curves), 3) 7 families of 32 equally spaced BPMs (magenta curves) and data from all 224 BPMs (light blue curves).

computed by the analysis of measurements from multiple BPMs, as a function of the number n of turns. From the results of this figure we see that tunes computed by the traditional technique of analyzing the measurements of just one BPM per turn (red curves) converge slowly to the real tunes, while considering more measurements per turn, results to faster convergence rates. From the comparison of results retrieved from measurements of symmetric BPMs (green and dark blue curves), it is shown that increasing the number of data points per turn, results to a more accurate tune determination. We also point out that analyzing data from equally spaced BPMs not having the same optics (magenta curves), or even from many BPMs that are not even equally spaced (light blue curves), a fast and accurate tune determination is obtained mainly because, the number of measurements per turn is considerably increased.

In the case of measurements obtained by not equally spaced BPMs, although the data are analyzed as if they were acquired from symmetric BPMs, the real trajectory is a function $f(s)$ of distance s and the data are considered to be measured at equally spaced positions s_i, with i being the BPM's index. Practically this means that a frequency analysis of the function $f_1(s_i) = f(s_i - d_i)$ is performed, whose Taylor expansion gives $f(s_i - d_i) \approx f(s_i) - f'(s_i)d_i + (1/2)f''(s_i)d_i^2 + \ldots$. Since the derivatives of $f(s_i)$ have the same fundamental frequency ν with $f(s_i)$, the frequency spectrum of f_1 contains ν, as well as combinations of ν with the frequency of d_i (Fig. 3) which is equal to 1 (or a multiple of 1), since the period of d_i is the total length of the ring which is considered as unity. Since the frequency analysis algorithm computes the fractional part of the real frequency, the addition of multiples of 1 does not, in general, alter the computed tunes.

EXPERIMENTAL DATA

The same analysis presented in the previous section was conducted in experimental data from the ESRF storage ring. The BPM system is based on multiplexed signal capable of giving a “pseudo” turn by turn data after averaging but with a high precision of a few μm in the position measurement [4]. Among the 224 existing BPM, 10 are used for other purposes and are not available. In Fig. 5, we display the difference of the horizontal and vertical tunes as computed by frequency analyzing all 214 BPMs with the values estimated by the standard turn-by-turn tune measurement. The various curves correspond to kicks of increasing horizontal amplitude (from 1 to 10mm) and a constant vertical one. The convergence to the correct tune is quite rapid and within the first 20 to 30 turns the tune difference is below 10^{-4}, which is remarkably good considering the experimental nature of the data. Note that the analysis of symmetrically positioned BPMs gives similar results, although the super-periodicity of the lattice and thus the optics functions is perturbed by linear imperfections. This is shown in Fig. 2b, where measured horizontal (red) and vertical (blue) beta functions are presented as estimated by a response matrix analysis. The experimental data (circles) are superimposed with the theoretical curves of the perfect model and displayed along one super-period for the sake of comparison. In this case, apart from the modulation in the azimuthal position, an amplitude modulation is added which indeed is 1-periodic and thus has the same effect, i.e. adds up in the spectrum frequencies which are multiples of 1.

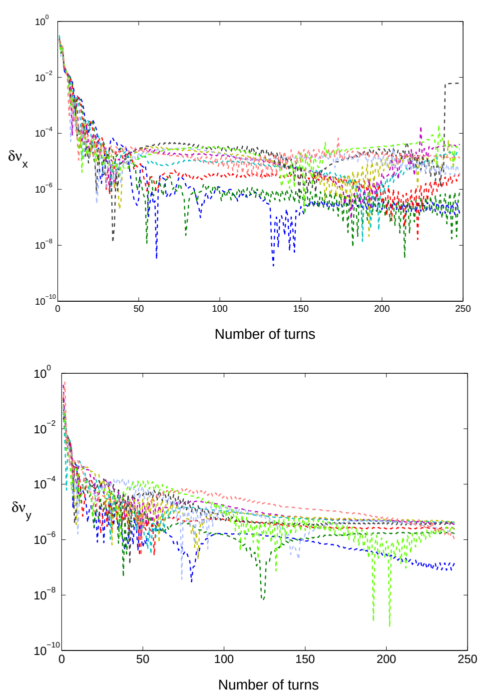

Figure 5: Convergence of the (top) horizontal ν_x and (bottom) vertical ν_y frequencies, after analysis of all 214 available ESRF BPMs, to the real tune values as a function of the number of turns n. The different curves correspond to different horizontal kicks and the same vertical one.

In the case of the CERN Super Proton Synchrotron (SPS), turn-by-turn positions have been measured at 26GeV using the LHC type bunches and the 1000-turn measurement system [8]. In Fig. 6, we present the convergence of the vertical tune to the real one, after analyzing the data from around 80 of the total 114 BPMs available around the ring. The different curves correspond to 4 vertical kicks applied with the tune meter kicker, ranging from 2 to 8mm and no horizontal one. The signal from around 30% of the BPMs were not taken into account, as it was found to suffer from noise due to timing problems, making the analysis even more challenging. In the case of the SPS data, the beam decoherence is much slower but the BPM resolution is not as good as in the case of the ESRF storage ring, corresponding to a few tens of microns. On the other hand, it is still remarkable that for all the different kicks, the real frequency is recovered with a precision of around 10^{-4}, within the first 50 turns.

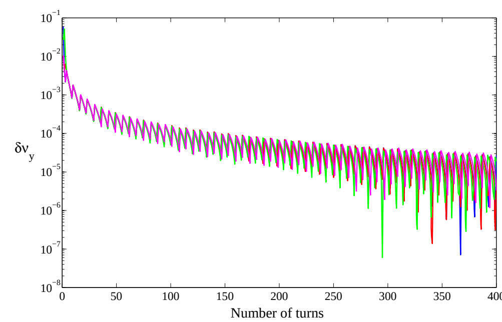

Figure 6: Convergence of the vertical ν_y frequencies, after analysis of around 80 of the available BPMs of the SPS, to the real tune values as a function of the number of turns n. The different curves correspond to different vertical kicks.

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